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CSIS 213-3941

Exam 3 Part 2

**Question 2**

 A relation S is defined on the set of integers.

For all a, b ∈  Z,  a S b  ⇔  8 | (a – b).

Prove (or disprove) that the relation is an equivalence relation.

**Proof:**

The relation S is an equivalence relation if, and only if, it is reflexive, symmetric, and transitive.

**S is reflexive:**

Suppose a is any integer. Since a – a = 0 and 8 | 0 (8 \* 0 = 0), then 8 | (a – a).

**S is symmetric:**

Suppose a and b are any integers such that a S b. By definition of S, this means that 8 | (a – b), and so, by definition of divisibility, a – b = 8k for some integer k. We know b – a = -(a – b). By substitution, b – a = -(8k). It follows that 8 | (b – 1) by definition of divisibility (-k is an integer), and thus b S a by definition of S.